SYNOPSIS OF THE THESIS

Vendor-Buyer Supply Chain Management models with and without backorder - an inspection at vendor site

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In

MANAGEMENT

By

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CONTENTS

1. Introduction 1
2. Research Motivation 4
3. Literature Review 5
4. Research Gap 34
5. Research Problem 34
6. Research Objective 35
7. Assumptions / Hypothesis 35
8. Scope of the Research 37
9. Research Methodology 37
10. Data Analysis 39
11. Finding and Conclusions 39
12. Contributions 40
13. Limitations and Scope for future research 42
14. Keywords 42
15. Bibliography 43

1. Introduction

Every business has to maintain some sort of inventory and inventory management becomes it's one of the important activities. The major activities of inventory management are the flow of goods and flow of information. The goods move down-stream from the supplier to the manufacturer, the manufacturer to the buyer (the dealer) and the buyer to the consumer. Information, initially generated from the customer in the form of demand, flow up-stream to the buyer (dealer), from the buyer to the manufacturer and finally from the manufacturer to suppliers of raw materials.

Industries need to cut the cost of their operation because of tough competition in the market. The inventory management is not just storing goods in an inventory but it deals with scheduling production of items, location of inventories, when to order, how much to order, how much quantities of items have to be shipped, frequency of shipments and many more with an objective to minimize the total expected cost of these operations. Research work done in inventory management tried to reduce the total expected cost of inventory management and guided industries to plan their activities accordingly to reduce cost. The experiment of JIT (Just in Time) is one example of this type of activity where inventory management plays an important role.

Ford Whitman Harris (1913) had done first research on inventory management and had given the first basic formula for Economic Order Quantity.

$$Q^*(EOQ) = \sqrt{\frac{2dK}{h}}$$

Where d : Annual demand
K : Ordering cost
h : Holding cost

(Goyal,1977) had initiated research work on integrated inventory management by giving “An integrated inventory model for a single supplier single customer problem”. The research work was followed by (Banerjee,1986) “A joint economic-lot-size model for purchaser and vendor” who had given Joint Economic-Lot-Size (JELS). (Lu, 1995) discussed one vendor multi-buyer inventory management. (Salameh&Jaber,2000) had discussed inventory management with imperfect production quality. (Cárdenas-Barrón,2000) had corrected (Salameh&Jaber,2000) formula for calculation of economic order quantity (EOQ) and (Wee et al.,2007) had further extended (Salameh&Jaber,2000) work by considering permissible shortage backordering. (Khan et al.,2011) extended (Salameh & Jaber,2000) research work by considering the inspection process errors. (Hsu & Hsu,2012b) had extended (Wee et al.,2007) model and gave inventory management for imperfect production quality and imperfect inspection with shortage backordering.

Inspection of items had been conducted by the buyer after the arrival of a fresh lot of items as mentioned in earlier research works. The imperfect production process could produce some defective items during the production. The buyer, after the arrival of lot, conducts 100% inspection of items, before the items are sold in the market for filtering out defective items.

The research work in this thesis is for imperfect production quality and imperfect inspection process. The inspection process is not done by the buyer as mentioned in earlier research work. The inspection process has been done by the vendor along with the production of items. It is assumed that the rate of inspection is greater than the production of items, the inspection process also finished just after the end of the production of items. There is no extra delay due to the inspection process. As the inspection process is also imperfect, it may classify non-defective items as defective (type I inspection error) and defective items as non-defective (type II inspection error). Because of type II inspection error, some defective items classified as non-defective item, could be sold in the market. After detection of defects consumer return back (sales return) defective items to the buyer (here the dealer) and get a replacement with a non-defective item. Considering these assumptions following models have been developed. These models are tested and compared with the help of numerical example using the same numerical values that had been consistently used by earlier related inventory management research works.

- Integrated model where backorder has not allowed
- The Buyers independent decision where backorder has not allowed
- Integrated model where backorder has allowed
- The Buyers independent decision where backorder has allowed

2. Research Motivation

Worldwide organizations are looking for cost-cutting by the management of its operations. Organizations that are manufacturing non-perishable items are coordinating manufacturing schedules, supply of items and inventories to reduce their operational cost. Optimization of supply chain management helps in cutting down operational costs.

The researches in the optimization of supply chain management have motivated this research to contribute to reduction of the operational cost of an organization. There is a lot of scope of research in the area of supply chain management.

Supply chain management research is a cross-platform research, based on mathematical modeling which includes Computer Science, Economics, Management and Mathematics. Mathematics is used for optimization of operational cost and guide production and shipment quantity and schedule (Economics and Management). The flow of information (from consumer to manufacturer and finally to the chain of suppliers) and the monitoring of levels of inventory items on a continuous or periodic basis involves Computer Science.

3. Literature Review

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
1	An integrated inventory model for a single supplier single customer problem.	Research Paper *International Journal of Production Research*	Goyal, S. K.	1977	(Goyal,1977) developed integrated model for single customer and single supplier and compared results of integrated solution with individual solutions. This model was first integrated model. **Figure 3.1** **Costs of integrated model** $\text{Cost to the supplier} = Z \cdot V(C(t^*), S(t^* \cdot K(t^*)))$ $\text{Cost to the customer} = (1-Z)V(C(t^*), S(t^* \cdot K(t^*)))$ $\text{where } Z = \frac{V(S(t_0 \cdot K(t_0)))}{V(S(t_0 \cdot K(t_0))) + V(C(t_0))}$ Source: (Goyal,1977) Where for customer => D = demand per unit of time, R = Cost of purchase order, h1 = customer stock holding cost per unit per unit time, t = time interval between successive order, V(C(t)) = Variable cost unit of time, for supplier => M = Setup cost, h2 = suppler stock holding cost per unit per unit time, T = time interval between successive set-up, K = T/t a positive integer, V(S(tK)) = variable cost per unit of time.	(Goyal,1977) models was simple model and did not consider rate of production.
2	A joint economic-lot-size model for purchaser and vendor.	Research Paper *Decision sciences*	Banerjee, A.	1986	(Banerjee,1986) developed Joint Economic-Lot-Size (JELS) and Joint Total Relevant cost (JTLC) for supply chain management with the assumption that the vendor produce products as per the order received from the purchaser on a lot-for-lot basis under deterministic condition, (Banerjee,1986).	It is not realistic for the vendor to produce item lot-for-lot each time after receiving an order

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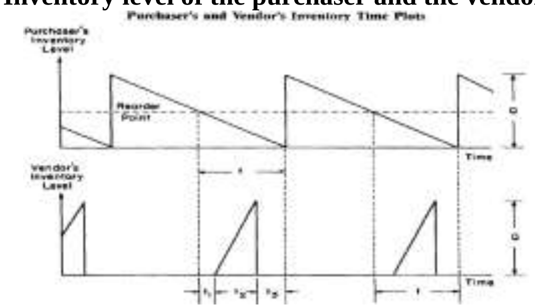
					Figure 3.2 shows inventory levels of the purchaser and the vendor over time. At reorder point the purchaser placed order, after t_1 time production started by the vendor which continue for t_2 time. After completion of production the lot of items shipped to the purchaser which reached to the purchaser in t_3 time. Thus $t = t_1 + t_2 + t_3$. **Figure 3.2** **Inventory level of the purchaser and the vendor**  Source: (Banerjee,1986) The formulation of costs for the purchaser and the vendor is shown in the Figure 3.3.	from the purchaser. The vendor may produce more items and send items in future. It is simple model and did not consider realistic assumptions like imperfect production quality, stochastic inventory etc.

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<tr><th>Srl. No</th><th>Title of paper</th><th>Literature type</th><th>Author</th><th>Publishing Year</th><th>Contribution</th><th>Gap / Future work</th></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td>
Figure 3.3
Summary of costs for individual optimal policies
Summary of Relevant Costs and Individual Optimal Policies
<table>
<tr><th></th><th>Purchaser</th><th></th><th>Vendor</th><th></th></tr>
<tr><td>General cost function</td><td>$TRC_p(Q) = \frac{DA}{Q} + \frac{Q}{2} rC_p$</td><td>(1)</td><td>$TRC_v(Q) = \frac{DS}{Q} + \frac{DQ}{2P} rC_v$</td><td>(4)</td></tr>
<tr><td>Economic lot size</td><td>$Q_p^* = \sqrt{[\frac{2DA}{rC_p}]}$</td><td>(2)</td><td>$Q_v^* = \sqrt{[\frac{2PS}{rC_v}]}$</td><td>(5)</td></tr>
<tr><td>Minimum total cost</td><td>$TRC_p(Q_p^*) = \sqrt{[2DArC_p]}$</td><td>(3)</td><td>$TRC_v(Q_v^*) = D\sqrt{[2SrC_v/P]}$</td><td>(6)</td></tr>
</table>
Note: $TRC_p(Q)$ = purchaser's annual total relevant cost for any lot size Q, $TRC_v(Q)$ = vendor's annual total relevant cost for any lot size Q, Q_p^* = purchaser's economic lot size (ELS), Q_v^* = vendor's economic lot size (ELS).
Source: (Banerjee,1986)
The formulation of costs for the joint policy is shown in the Figure 3.4.
Figure 3.4
JTRC and JELS – Q_i*.
$JTRC(Q) = {}^D/_Q(S+A) + {}^Q/_2 r({}^D/_P C_v + C_p).$ $Q_j^* = \sqrt{[\frac{2D(S+A)}{r({}^D/_P C_v + C_p)}]}.$
Source: (Banerjee,1986)
Where D = annual demand, S = setup cost for the vendor, A = ordering cost per order for the purchaser, r = annual inventory carrying charges, C_v = unit production cost occurred to the vendor, C_p = unit purchase cost to the purchaser and Q = order or production lot size in units
</td><td></td></tr>
<tr><td>3</td><td>Determination of Production Cycle and Inspection Schedules in a</td><td>Research Paper

Management Science</td><td>Lee, H. L. & Rosenblatt, M. J.</td><td>1987</td><td>(Lee&Rosenblatt,1987) considered production of single item on single machine where at the beginning of production, the production process is in an “in-control” state. It produced</td><td>During production process machine goes</td></tr>
</table>

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
	Production System.				perfect quality items with negligible number of defective items. As time goes the production process deteriorates and shifted to “out-of-control” state and produced defective and sub-standard items. (Lee&Rosenblatt,1987) assumed that the production process remain in “in-control” state for a random time duration which is exponentially distributed with mean 1/μ. (Lee&Rosenblatt,1987) assumed that inspections of the production process were carried at end of each production run. If the production process was found in “out-of-control” state, a restoration work was carried out at some cost. At start of each production cycle, production is in “in-control” state. Using above assumptions (Lee&Rosenblatt,1987) tried to derive Economic Manufacturing Quantity for a production cycle and inspection schedule. **Figure 3.5** **The optimal production run duration T for n inspections per run** $T^{*}(n) = \left[\frac{2(K + nv)D}{P(P - D)h + D(s\alpha P/\mu - r)\mu^{2}/n}\right]^{1/2}$ Source: (Lee&Rosenblatt,1987) Where D = Demand rate, P = Production rate, T = Cycle time	for wear and tear and it impact on the quality of item produced. (Lee&Rosenblatt,1987) had tried to find impact of the wear and tear on production. (Lee&Rosenblatt,1987) discussed production of imperfect quality items during the production. They did not focus much on impact on supply chain.

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					for production lot, K = Setup cost, s = cost incurred by producing a defective item (rework, repair, replacement, loss of goodwill, etc.), α = percentage of defective units, υ = cost of inspecting the production process, r = cost of restoring the production process, n = number of inspections per production run, n ≥1, T_i = elapse time from beginning of production run until the i^{th} inspection.	
4	A joint economic-lot-size model for purchaser and vendor: a Comment	Research Paper *Decision sciences*	Goyal, S. K.	1988	(Goyal,1988) generalized (Banergee,1968) model by removing lot-for-lot policy, assumed that the vendor may produce an integer multiple of order lot quantity and supply multiple lots from a production run. The economic order quantity (EOQ) obtained by him is shown in the figure 3.6 **Figure 3.6** **Economic Order Quantity (EOQ) for the purchaser** $Q(n)=\left(\frac{2D(A+\frac{S}{n})}{r(C_Q-C_v+nC_v(1+\frac{D}{P}))}\right)^{1/2}$ Source: (Goyal,1988) He calculated joint total relevant cost (JTRC) for the vendor and the purchaser. The JTRC is given in figure 3.7 **Figure 3.7** **Joint Total Relevant Cost (JTRC)** $JTRC(n)=[2Dr(A+\frac{S}{n})(C_Q-C_v+nC_v(1+\frac{D}{P}))]^{1/2}$ Source: (Goyal,1988) The optimal value of n* is calculated using the condition	(Goyal,1988) proposed multi shipment policy in inventory management and showed that it was better than lot-to-lot production policy. This model becomes basic model for future models

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					given in the figure 3.8 **Figure 3.8** **The optimal value condition for n*** $n^*(n^*+1) \geq \frac{S(C_Q - C_v)}{AC_v(1+\frac{D}{P})} \geq n^*(n^*-1)$ Source: (Goyal,1988) Where D = annual demand, S = setup cost for the vendor, A = ordering cost per order for the purchaser, r = annual inventory carrying charges, C_v = unit production cost occurred to the vendor, C_Q = unit purchase cost to the purchaser, Q = order or production lot size in units and n = multiple of order such that production quantity = nQ.	
5	An integrated JIT inventory model	Research Paper *International Journal of Operation & Production management*	Banerjee, A. & Kim, S. L	1995	As per (Banerjee&Kim,1995) in the Just in Time production system a buyer may order some fixed quantity Q at a regular interval of time. They pointed out that if the vendor (manufacturer) produced ordered items for a lot-for-lot policy then there would be production set-ups for each Q quantity produced, which is very frequent. They suggested it would be more economical that the vendor will produce NQ items in one production lot and send N number of Q items to the vendor and the same time raw material supplier also supply raw materials at regular intervals. The inventory levels for the buyer, the vendor (finished items and raw material) is explained in the figure 3.9	(Banerjee&Kim,1995) had discussed lot-to-lot production for JIT and showed inventory levels for the vendor and the buyer.

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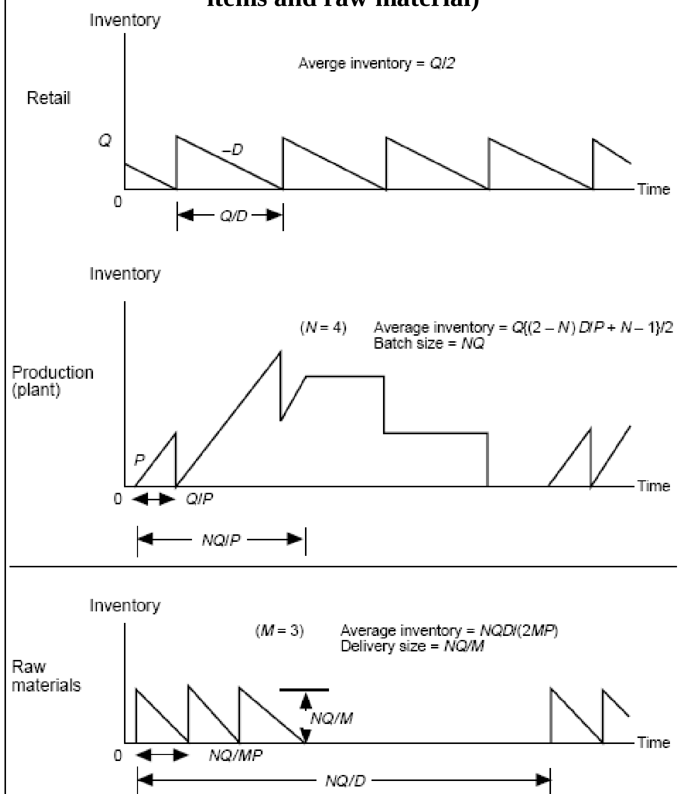
<tr><td></td><td></td><td></td><td></td><td></td><td>

Figure 3.9

The inventory levels for the buyer, the vendor (finished items and raw material)



Source: (Banerjee&Kim,1995)

As per (Banerjee&Kim,1995) model optimal Q* in Just-In-Time could be obtained formulas given in figure 3.10

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					Figure 3.10 **Calculation of Q* in JIT** $Q^* = (\alpha / \beta)^{\frac{1}{2}}$ where $\alpha = 2D[(A_m M + S)/N + A_r]$ and $\beta = Nh_m D/(MP) + h_p[(2 - M)D/P + N - 1] + h_r$ Source: (Banerjee&Kim,1995) With optimal condition for M* (figure 3.11) and condition for N* (figure 3.12) **Figure 3.11** **Condition for M* in JIT** $M^*(M^* - 1) \leq \delta/\phi \leq M^*(M^* + 1)$ Source: (Banerjee&Kim,1995) **Figure 3.12** **Condition for N* in JIT** $N^*(N^* - 1) \leq \eta/\theta \leq N^*(N^* + 1)$, where $\eta = (A_m M + S)\{h_p(2D/P - 1) + h_r\}$ and $\theta = A_r(D/P)\{h_m/M + h_p(1 - D/P)\}$. Source: (Banerjee&Kim,1995) Where A_m = raw material ordering cost, A_r = Supplier's order processing and shipment cost, D = demand rate, h_m = raw material holding cost, h_p = finished goods inventory holding cost, h_r = inventory holding cost for the buyer, M = raw material lot size factor Q_m = NQ/M, N = production lot size factor Q_p = NQ, Q delivery lot size, S = production set-up cost.	
6	A one-vendor multi-buyer	Research Paper	Lu Lu	1995	(Lu,1995) minimize the vendor's total annual cost for single	Tried to minimize only

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	integrated inventory model.	*European Journal of Operational Research*			vendor and single buyer with subject to maximum cost that the buyer is ready to pay where the vendor has advantage over the buyer in purchase negotiation and know the buyer's annual demand and order frequency in advance. (Lu,1995) also had given heuristic approach to minimize the vendor's total annual cost for single vendor and multiple buyers. (Goyal,1988) model had an assumption that the vendor will only supply items to purchaser after completion of entire production lot. (Lu,1995) had relaxed this assumption in this research work.	the vendor's total annual cost in the integrated model.
7	A one-vendor multi-buyer integrated inventory model: A comment.	Research Paper *European Journal of Operational Research*	Goyal, S. K.	1995	(Goyal,1995) extends work done by (Goyal,1988) and (Lu,1995) and given an approach which is capable of giving better relevant total costs of the single vendor-single purchaser production-inventory systems. (Goyal,1995) had taken ratio of $(i+1)^{th}$ shipment to i^{th} shipment equal to n. Economic Order Quantity, EOQ for k number of lots per production is given in the figure 3.13 and minimum joint total annual cost is given in figure 3.14. **Figure 3.13** **The Economic Order Quantity for k lots/production** $q(k) = \sqrt{\frac{2D(S+kA)(n^2-1)}{r(n^{2k}-1)\left(C_Q + \frac{C_v}{n}\right)}}$ Source: (Goyal,1995)	Extended by (Goyal,1988) and (Lu,1995) research and derived better relevant total costs of the single vendor-single purchaser production-inventory systems

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					Figure 3.14 **The Minimum Joint Total Annual Cost** $$\mathrm{JTRC}(q(k)) = \sqrt{\frac{2rD(C_Q + C_v/n)(n-1)(n^k+1)(S+kA)}{(n+1)(n^k-1)}}$$ Source: (Goyal,1995) Where JTRC = Joint annual Total Relevant Cost, r = vendor's annual rate of production, P = vendor annual rate of production, S = vendor's setup cost per setup, C_v = vendor's unit manufacturing cost, Q = production lot quantity per production, k = number of shipments per production, D = annual demand rate, P = vendor annual rate of production, n = P / D, C_Q = unit purchase price paid by purchaser, q_i = size of i^{th} shipment	
8	On an inventory model with deteriorating items decreasing time-varying and shortages.	Research Paper *European Journal of Operational Research*	Benkherouf, L.	1995	(Benkherouf,1995) had given optimal replenishment policy for items that are continuously deteriorating over time at a constant rate and demand rates are deceasing over known time period with shortage in inventory is allowed. Perishable items like food stuff, medicines, volatile liquids, blood banks, etc. are considered in this research work.	(Benkherouf,1995) discussed inventory policies for perishable items.
9	The single-vendor single-buyer integrated production-inventory model with a generalised policy.	Research Paper *European Journal of Operational Research*	Hill, R. M.	1997	As per (Hill,1997) none of policies given by (Lu,1995) and (Goyal,1995) was have optimal solution. As per him optimal solution could be obtained when successive shipment quantities within a production batch should increased by a fixed factor. The first shipment quantity q* is given in figure 3.15	(Hill,1997) discussed (Lu,1995) and (Goyal,1995) models and pointed out these models are not giving optimal solutions.

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					and mean total cost for q* is given in figure 3.16. **Figure 3.15** **The first shipment quantity** $q^* = \left(\frac{2(A_1 + nA_2)D\lambda(\lambda^2 - 1)}{(h_1 + \lambda h_2)(\lambda^{2n} - 1)} \right)^{1/2}$ Source: (Hill,1997) **Figure 3.16** **The mean total cost incurred by the system** $C(q^*) = 2\left(\frac{(A_1 + nA_2)(h_1 + \lambda h_2)D}{2} \cdot \frac{(\lambda - 1)(\lambda^n + 1)}{\lambda(\lambda + 1)(\lambda^n - 1)} \right)^{1/2}$ Source: (Hill,1997) Where A_1 = the fixed production setup cost, A_2 = the fixed order/shipment cost, h_1 = the stockholding cost for the vendor, h_2 = the stockholding cost for the buyer, D = the demand rate, P = the production rate for the vendor, n = the number of shipments per production run, q = the size of first shipment, λ = the proportional increase in the size of successive shipments, C = the mean cost incurred by the system per unit time, P > D and h_2 > h1.	
10	Optimal strategy for the integrated vendor-buyer inventory model	Research Paper *European Journal of Operational Research*	Vishwanathan, S	1998	(Vishwanathan,1998) discussed two replenishment strategies for integrated vendor-buyer inventory model. The first strategy replenished the buyer's inventory with equal quantity items each time. The second strategy replenished the vendor's inventory with available inventory of an item so that after receiving items the buyer's inventory reached to maximum for the item received.	As per (Vishwanathan,1998) equal quantity replenishment is better when the vendor's holding cost is lower than the buyer's.

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					(Vishwanathan,1998) observed, when there is high ratio value of holding cost of the buyer to holding cost of the vendor, the first strategy of equal item replenishment is more attractive. Higher production rate with respect to demand rate gave less overall cost	This concept was used by latter research works where equal quantity replenishment strategy had been used
11	An optimal policy for a single-vendor single-buyer integrated production-inventory system with capacity constraint of the transport equipment.	Research Paper *International Journal of Production Economics*	Hoque, M. A. & Goyal, S. K	2000	(Hoque&Goyal,2000) had developed optimal policy for a single-vendor, single-buyer integrated production system with equal and unequal size batch shipment between stages and limited capacity to transport items.	(Hoque&Goyal,2000) had examine equal and unequal size shipment lots
12	On optimal two-stage lot sizing and inventory batching policies.	Research Paper *International Journal of Production Economics*	Hill, R. M.	2000	(Hill,2000) had discussed coordination between two successive stages of multi stage production system. He had classified problem as follows **Production rate:** greater than or less than between stages **Production batch size**: greater than or less than between stages. **Items transfer type:** continuous or in batches He had also observed that equal size batches had given better result.	(Hill,2000) examined coordination between multi stage production system
13	Determination of economic production-shipment policy for	Research Paper *European Journal of*	Goyal, S. K. & Nebebe, F.	2000	According to (Goyal&Nebebe,2000) had developed a model for single vendor single buyer. The suggested that the first	(Goyal&Nebebe,2000) worked on lead time

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	a single-vendor-single-buyer system	*Operational Research*			shipment size should be smaller than rest shipment size and equal to (Rate of product/Rate of demand). It would ensure quick delivery after receiving an order and rest (n-1) shipments would be of equal size. They tried to provide simple alternative policy to determine optimal batch quantity for the vendor, economical number of shipments sent from the vendor to the buyer and economical size of shipments. The annual cost for the vendor-buyer had been given as **Figure 3.17** **The total annual cost of the vendor-buyer** $$C(n,q) = \frac{(A_1 + nA_2)D}{q(1+(n-1)x)} + \frac{q}{2}\left[h_1 \frac{(2D + (P-D)(1+(n-1)x))}{P} + \frac{(h_2 - h_1)(1+x^2(n-1))}{(1+x(n-1))}\right].$$ Source: (Goyal&Nebebe,2000) Where A_1 = production Set-up cost, A_2 = shipment cost, h_1 = the vendor's holding cost, h_2 = buyer's holding cost, D = annual demand rate, P = production rate, x = ratio of production rate to demand rate (i.e., P/D), q = size of first shipment. N = number of shipments, C = total annual cost	and tried to reduce by reducing size of first shipment lot.
14	Economic production quantity model for items with imperfect quality.	Research Paper *International Journal of Production Economics*	Salameh, M. K. & Jaber, M. Y.	2000	(Salameh&Jaber,2000) extended the traditional EPQ/EOQ model by taking consideration of imperfect quality items. They considered that 100% screening of items and poor quality would be sold at end of screening process.	(Salameh&Jaber,2000) did not considered the fact there may be also some error in screening process. Backorder in

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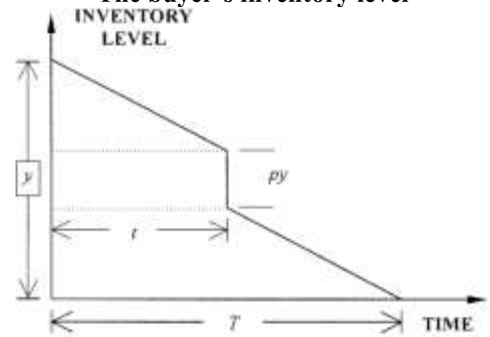
					Figure 3.18 **The buyer's inventory level**  Source: (Salameh&Jaber,2000) The Economical Order Quantity suggested by (Salameh&Jaber,2000) is given in figure 3.19. **Figure 3.19** **The Economical Order Quantity** $y^* = \sqrt{\frac{2KD\,\mathrm{E}[1/(1-p)]}{h[1-\mathrm{E}[p]-D(1-\mathrm{E}[1/(1-p)])/x]}}$ Source: (Salameh&Jaber,2000) Where y = order size, K = ordering cost, p = percentage of defective items, x = screening rate, D = demand rate per year, h = holding cost	inventory was also not considered.
15	Observation on: " Economic production quantity model for items with imperfect quality".	Research Paper *International Journal of Production Economics*	Cárdenas-Barrón L. E.	2000	(Cárdenas-Barrón,2000) found an error in (Salameh&Jaber, 2000) EOQ formula (given in figure 3.19) and gave the corrected formula as given in figure 3.20.	

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					Figure 3.20 **The Economical Order Quantity** $$y^* = \sqrt{\frac{2KDE[1/(1-p)]}{h[1-E[p]-(2)(D/x)(1-E[1/(1-p)]]}}$$ Source: (Cárdenas-Barrón,2000) Where y = order size, K = ordering cost, p = percentage of defective items, x = screening rate, D = demand rate per year, h = holding cost	
16	Recent trends in modeling of deteriorating inventory	Research Paper *European Journal of Operation Research*	Goyal, S. K. & Giri, B. C.	2001	(Goyal&Giri,2001) had classified inventory items into following three categories 1. Obsolescence 2. Deterioration 3. No Obsolescence/Deterioration Obsolescence items are those items which loosed their values due to change in technology or introduction of new product. For example spare parts of an aircraft which has been replaced by new advance aircraft. These spare parts loosed its value. Deterioration items are those items that have very short life and after that they loosed their value. These items are also referred as perishable items. For example foodstuff, green vegetables, human blood, medicine with expiry date are fall into deterioration items categories. (Goyal&Giri,2001) had discussed inventory models	

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					dealing with deterioration items.	
17	Quality improvement and setup reduction in the joint economic lot size model.	Research Paper *European Journal of Operation Research*	Affisco, J. F., Paknejad, M. J. & Nasri, F.	2002	(Affisco et al.,2002) had discussed co-maker concept in which the supplier and purchaser are value chain partner in a manufacturing process. They discussed following three different cases 1 The basic model as given by (Banerjee,1986) 2 Quality Improvement 3 Simultaneous quality improvement and setup cost reduction (Affisco et al.,2002) extended the basic model of (Banerjee,1986) and suggested that the purchaser could go for 100% inspection if the inspection cost is less than the cost of selling defective items. (Affisco et al.,2002) discussed quality improvement of manufacturing process by some investment with an objective to minimize joint total relevant cost (JTRC) and get joint economic lot size (JELS). (Affisco et al.,2002) also discussed simultaneous quality improvement and setup cost reduction. The setup cost reduction allowed smaller JELS. (Affisco et al.,2002) suggested that there should be a continuous quality improvement program and setup cost reduction could be taken as complementary program in manufacturing process	Discussed concept of co-maker in production system and its impact on inventory management

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18	An integrated vendor-buyer cooperative inventory model for items with imperfect quality.	Research Paper *Production Planning & Control: The Management of Operations*	Huang, C. K.	2002	(Huang,2002), tried to develop a model to determine an optimal integrated vendor–buyer integrated policy for just-in-time (JIT) environment with an aim to minimize the total annual cost incurred by the vendor and the buyer. The model also taken account of imperfect quality items.	
19	Note on: economic production quantity model for items with imperfect quality–a practical approach.	Research Paper *International Journal of Production Economics*	Goyal, S. K. & Cárdenas-Barrón L. E.	2002	(Goyal&Cárdenas-Barrón,2002) extended (Salameh&Jaber,2000) model and developed a simple approach for determining the economic production quantity for an item with imperfect quality through their technical note. The simplified formula is given in figure 3.21. It could be compared with formula given by (Salameh&Jaber,2000) and latter modified by (Cárdenas-Barrón,2000) in the figure 3.20 given above **Figure 3.21** **The Economical Order Quantity** $y = y^{**} = \sqrt{\frac{2KDE[1/(1-p)]}{h}}$ Source: (Goyal&Cárdenas-Barrón,2002)	Extended (Salameh&Jaber,2000) by Adopting Simple approach for EOQ
20	The economic production lot-sizing problem with imperfect production processes and imperfect maintenance.	Research Paper *International Journal of Production Economics*	Ben-Daya, M.	2002	(Ben-Daya,2002) developed an integrated model to determine Economics Production Quantity (EPQ) and Preventive Maintenance (PM) level for imperfect production process having a deterioration distribution and increasing hazard rate. He found that performing preventive maintenance (PM)	Author had given more focus on preventive maintenance (PM) of manufacturing equipments rather than

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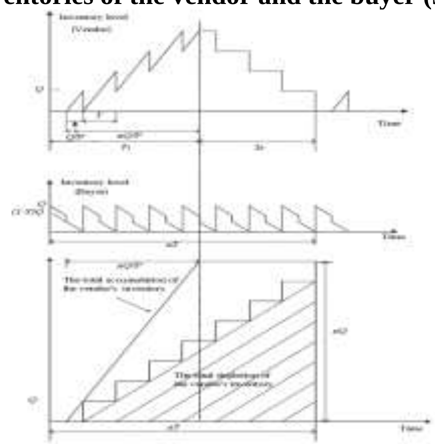
					results into reduction quality related cost. As per (Ben-Daya,2002) when preventive maintenance (PM) cost become higher than reduction quality related cost, further preventive maintenance (PM) is not justified.	inventory management
21	An optimal policy for a single-vendor single-buyer integrated production-inventory problem with process unreliability consideration.	Research Paper *International Journal of production economics*	Huang, C. K.	2004	(Huang,2004) tried to get optimal policy for a single-vendor single-buyer integrated production with process unreliability for Just-in-time (JIT). According to (Huang,2004) in JIT the buyer had a problem to know how much quantity could be ordered and the vendor had problems to know economic production batch quantity and number of shipments per order. The inventories of the vendor and the buyer model of is given in figure 3.22 **Figure 3.22** **Inventories of the vendor and the buyer (JIT)**  Source: (Huang,2004)	Inventory Management for JIT.

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					The condition to find optimal number of shipments per lot as per (Huang,2004) is given in figure 3.23 **Figure 3.23** **The optimal number of shipments n* in (JIT)** $(n^*-1)n^* \leqslant \frac{(S_V+S_B)\{2DM((h_B/x)+(h_V/P))-h_V-2Dh_B/x+(1-E[Y])h_B\}}{(1-DM/P)Fh_V} \leqslant n^*(n^*+1)$ Source: (Huang,2004) Using n* as given in figure 3.23, (Huang,2004) given a formula as shown in the figure 3.24 to calculate the size of optimal shipment Q* from the vendor to the buyer. **Figure 3.24** **The optimal shipment quantity Q*** $Q^*=\sqrt{\frac{2D[(S_V+S_B)/n+F]M}{[2Dh_B/x-(n-2)Dh_V/P]M+(n-1)h_V-2Dh_B/x+h_B(1-E(Y))}}$ Source: (Huang,2004)	
22	Economic ordering quantity models for items with imperfect quality.	Research Paper *International Journal of Production Economics,*	Papachristos, S. & Konstantaras, I..	2006	(Papachristos&Konstantaras,2006) had pointed out that conditions for non-shortage of items, as mentioned in (Salamesh&Jaber,2000) and (Chan et al.,2003), did not really prevent occurrence of shortage in the inventory. (Papachristos&Konstantaras,2006) extended (Salamesh&Jaber,2000) model with modified condition.	Pointed there had shortage of items in (Salamesh&Jaber,2000) and (Chan et al.,2003)
23	Fuzzy economic production quantity model for items with imperfect	Research Paper *International Journal of*	Chen, S. H., Wang, C.C., & Chang S.	2007	(Chen et al.,2007) gave a Fuzzy Economic Production Quantity (FEPQ) model with imperfect products where	Fuzzy model for imperfect products

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	quality.	*Innovative Computing, Information and Control*	M.		defective items could be sold at a discount price. In the model cots and quantities represented in fuzzy numbers. They used Graded Mean Integration Representation method to defuzzing and Kuhn-Tucker conditions to find optimal economic production quantity. (Chen et al.,2007) had provided following equation as mentioned in figure 3.25 to calculate Optimal Production Quantity **Figure 3.25** **The optimal production quantity Q*** $Q^* = \sqrt{\frac{2(d_1k_1+2d_2k_2+2d_3k_3+d_4k_4)}{2p(d_1h_1+2d_2h_2+2d_3h_3+d_4h_4)+(1-p)^2(h_1+2h_2+2h_3+h_4)}}$ Source: (Chen et al.,2007) Where (h1,h2,h3,h4) = fuzzy holding cost, (k1,k2,k3,k4) = fuzzy setup cost, (d1,d2,d3,d4) = fuzzy demand, p = the percentage of defective items in a production lot,	
24	Optimal inventory model for items with imperfect quality and shortage backordering	Research Paper *Omega*	H. M. Wee, Jonas Yu and M. C. Chen	2007	This research work generalized production lot size model with backordering. It extended the approach of Salameh & Jaber (2000) by considering permissible shortage backordering and the effect of varying backordering cost values. It introduced the concept of backorder due to imperfect quality of production.	The backorder level reached to zero on arrival of fresh lot of items. In this research work inspection process is done at the buyer's site

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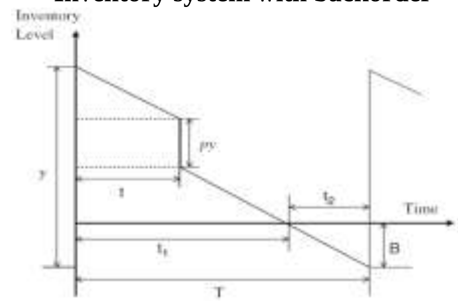
					Figure 3.26 **Inventory system with backorder**  Source: (Wee et al.,2007) **Figure 3.27** **Optimal Order Size y* and Optimal Back Order Size B*** $y^* = \sqrt{\frac{(2DK + B^2h + B^2b)E[1/(1-p)]}{h(1-E[p]-2(D/x)(1-E[1/(1-p)]))}},$ $B^* = \frac{h}{(h+b)(1/\beta-\alpha)\ln((1-\alpha)/(1-\beta))}.$ Source: (Wee et al.,2007) Where y = order size, D = demand rate, x = screening rate, K = ordering cost, B = maximum backorder quantity allowed, h = inventory holding cost, p = defective percentage, α = minimum value for p, β = maximum value for p.	after receiving a new lot of items
25	Economic order quantity for items with imperfect quality: revisited.	Research Paper *International Journal of Production Economics*	Maddah, B. & Jaber, M. Y.	2008	(Maddah&Jaber,2008) considered imperfect quality items and screening process as random function and analyzed (Salmesh&Jaber,2000) model using renewal theory. They found that effect of screening speed and variation in supply process due to random imperfect items, the order quantity	Used renewal theory

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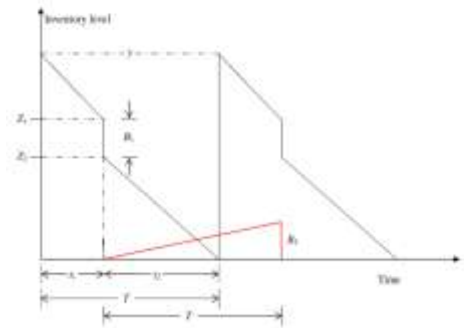
					calculated by them was larger than (Salmesh&Jaber,2000) model and the same profit was also found lesser. The optimal order quantity is given in figure 3.26. The optimal order quantity of (Salmesh&Jaber,2000) has been given in figure 3.19 which was corrected latter by (Cárdenas-Barrón,2000), figure 3.20. **Figure 3.28** **The Economical Order Quantity** $y^* = \sqrt{\dfrac{2KD}{h[E[(1-P)^2] + 2E[P]D/x]}}$ Source: (Maddah&Jaber ,2008) Where K = Ordering cost, D = Demand Rate, h = inventory holding cost, x = rate of inspection, y = order size, P = fraction of defective items in a lot.	
26	Exact closed-form solutions for ''optimal inventory model for items with imperfect quality and shortage backordering''	Research Paper *Omega*	Chang, H. C. & Ho, C. H.	2010	(Chang&Ho,2010) revisit (Wee et al.,2007) and apply the well-known renewal- reward theorem to obtain a new expected net profit per unit time (gives better result). They also provided an approach to solve the same problem algebraically from another direction.	Discussed imperfect product with shortage backorder
27	An economic order quantity (EOQ) for items with	Research Paper *International Journal of Production*	Khan, M., Jaber, M. Y. & Bonney, M.	2011	(Khan et al.,2011) extend the work of (Salameh&Jaber,2000) and introduce the concept that inspection of items for defects can also have errors. A defective item can be classified as	Discussed imperfect product with shortage backorder and

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
	imperfect quality and inspection errors	*Economics*			non-defective and non-defective item can also classify as defective. There are two types of inspection errors Type I Error: An inspector may classify a non-defective item as defective Type II Error: An inspector may classify a defective item as non-defective B2 defective items classified as non-defective would be sold in market and latter replaced and stored in inventory. **Figure 3.29** **Inventory Level over time**  Source: (Khan et el.,2011) (Khan et el.,2011) derived the formula for calculating the expected annual profit as given in the figure 3.30 and EOQ as given in the figure 3.31.	inspection error

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					Figure 3.30 **Expected annual profit** $E[TPU(y)] = sD + \frac{sDE[p]E[m_2]}{(1-E[p])(1-E[m_1])} + \frac{vDE[m_1]}{(1-E[m_1])} + \frac{vDE[p]}{(1-E[p])(1-E[m_1])} - \frac{D\left[\frac{K}{y} + c + d + c_r(1-E[p])E[m_1] + c_a E[p]E[m_2] + \frac{h}{2}\left\{\left(\frac{2}{x} - \frac{D}{x^2} + \frac{E[A^2]}{D}\right)y\right\}\right]}{(1-E[p])(1-E[m_1])} - h\frac{yE[p]E[m_2]}{2}$ Source: (Khan et el.,2011) **Figure 3.31** **Economic Order Quantity (EOQ)** $y^* = \sqrt{\frac{2KD}{hE[p]E[m_2](1-E[p])(1-E[m_1])+hD\left(\frac{2}{x}-\frac{D}{x^2}+\frac{E[A^2]}{D}\right)}}$ Source: (Khan et el.,2011)	
28	Disaggregation and consolidation of imperfect quality shipments in an extended EPQ model	Research Paper *International Journal of Production Economics*	Yassine, A., Maddah, B. & Salameh, M.	2012	(Yassine et al.,2012) discussed a tradeoff between disaggregation of imperfect quality items shipment and shipped multiple imperfect items during a production cycle vs. consolidation of imperfect quality items shipment over multiple production cycle where imperfect items for multiple production cycle. (Yassine et al.,2012) showed that disaggregation of imperfect quality items shipment reduced overall inventory management cost.	There is no change in assumptions as made by (Salameh&Jaber,2000)
29	A note on "Optimal inventory model for items with imperfect quality	Research Paper *International Journal of*	Hsu, J. & Hsu, L. (2012b)	2012	In (Wee et al.,2007) backorder get clear as soon as new batch of items arrived. It did not consider that inspection of items need some time. (Hsu&Hsu,2012) had corrected this problem and proposed a model	Discussed imperfect product with shortage backorder

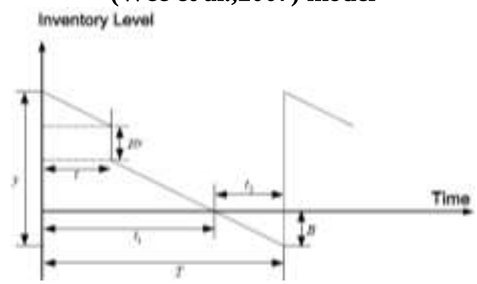
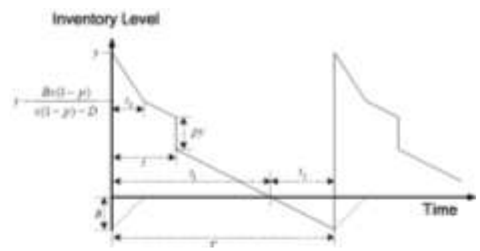
Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
	and shortage backordering"	*Industrial Engineering Computations*			**Figure 3.32** **Inventory system with complete backordering (Wee et al.,2007) model**  Source: (Hsu & Hsu,2012b) **Figure 3.33** **Behavior of the inventory level over time for the model corrected by (Hsu, J.& Hsu, L.,2012b)**  Source: (Hsu&Hsu,2012b) They found the economic order quantity (EOQ) and optimal backorder quantity allowed as shown in the figure 3.34 and expected total profit per unit time as given in figure 3.35	

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					Figure 3.34 **Economic Order Quantity (EOQ)** $y^{*}=\sqrt{\frac{2KD}{h\left\{E[(1-p)^{2}]-R^{2}A_{1}+2E[p]\frac{D}{x}\right\}-bR^{2}\left(1+A_{2}\frac{D}{x}\right)}}$ $B^{*}=y^{*}R$ Where $R=\frac{h(1-E[p]-A_{1}D/x+A_{2})}{2(hA_{1}+b+bA_{3}D/x)}$ Source: (Hsu&Hsu,2012b) **Figure 3.35** **Expected Total Profit per unit time (ETPU)** $ETPU(B,y)=sD+vD\frac{E[p]}{(1-E[p])}-\frac{KD}{(1-E[p])y}-\frac{cD}{(1-E[p])}-\frac{dD}{(1-E[p])}-\frac{1}{2}k\frac{DB}{x(1-E[p])}E\left[\frac{(1-p)}{(1-p-\frac{D}{x})}\right]$ $-\frac{1}{2}h\left(\frac{yE[(1-p)^{2}]}{(1-E[p])}-\frac{B}{(1-E[p])}E\left[\frac{(1-p)^{2}}{(1-p-\frac{D}{x})}\right]-B+\frac{B^{2}}{y(1-E[p])}E\left[\frac{(1-p)}{(1-p-\frac{D}{x})}\right]\right)-k\frac{E[p]yD}{x(1-E[p])}$ $-\frac{1}{2}bB^{2}\left(\frac{1}{y(1-E[p])}+\frac{D}{xy(1-E[p])}E\left[\frac{1}{(1-p-D/x)}\right]\right)$ Source: (Hsu&Hsu,2012b)	
30	Lot sizing in case of defective items with investments to increase the speed of quality control	Research Paper *Omega*	Hauck, Z., Vörös, J. (2015)	2015	(Hauck,2015) stated that increasing the speed of inspection process enables the system to respond fast and save money. (Hauck,2015) had developed two models. The first model always remain in the same state while in second model the percentage of defective items was different in consecutive lots and the same time speed of inspection of items was also different. (Hauck,2015) had stated that increasing speed of	(Hauck,2015) stared that impact of increasing inspection process speed need more research work and that would enable to get optimal

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					inspection process is controversial. Increasing the speed reduce total inventory management cost but when percentage of defective items was high and there was backlog of items then it increased total inventory management cost	inspection process speed.
31	Optimal Buyer's Replenishment Policy in the Integrated Inventory Model for Imperfect Items.	Research Paper *Mathematical Problems in Engineering*	Yueli, L., Jiangtao M. & Yucheng W.	2016	(Yueli&Yucheng,2016) extended (Maddah&Jaber,2008) model by making assumption that ordering cycle would be based on demand rate, number of items in a lot and mathematical exception for rate of defects in a lot. By adding these assumptions they had discussed possibilities of shortage of items due to random defective items in lots. They had taken two cases. For the first case, extra items were added in lots to avoid shortage for second case they let the shortage happen. For these conditions they tried to find optimal ordering cycle.	(Yueli&Yucheng,2016) pointed the possibility of shortage of items at buyer end as defective percentage is a random variable.
32	Integrated supply chain inventory model with quality improvement involving controllable lead time and backorder price	Research Paper *International Journal of Industrial Engineering Computations*	Jindal, P. & Solanki, A.	2016	(Jindal&Solanki,2016) had discussed continuous review inventory management and considered order quantity, reorder point, lead time, process quality and backorder price discount and number of shipments as decision variables and tried to minimize total related cost of inventory. (Jindal&Solanki,2016) had made assumption that buyer was motivating consumers to wait for possible backorder by giving price discount. They also assumed that items received from the vendor contain defective items. They tried to get	Discussed continuous review of inventory with order quantity, reorder point, lead time, process quality and backorder price discount and number of shipments as decision variables

Srl. No	Title of paper	Literature type	Author	Publishing Year	Contribution	Gap / Future work
					optimal values for decision variable by using iterative method to minimize total expected cost.	
33	Inventory Modeling for Imperfect Production Process with Inspection Errors, Sales Return, and Imperfect Rework Process.	Research Paper *International Journal of Mathematical, Engineering and Management Sciences*	Khanna, A., Kishore, A. & Jaggi, C. K.	2017	(Khanna et al.,2017) tried to minimize losses occurred due to production of defective items and proposed reworking on defective items to remove defects. They tried to consider human error is a reality of life and consider that the rework process was also imperfect. To improve consumer satisfaction they assumed 100% full price return to consumer on sales return due to manufacturing defects. They tried to maximize the expected total profit per unit time. The pertinence of the model can be found in most manufacturing industries like textile, electronics, furniture, footwear, crockery etc.	Proposed reworking of defective items for imperfect production and inspection error
34	An Integrated Imperfect Production–Inventory Model with Optimal Vendor Investment and Backorder Price Discount.	Research Paper *Information Technology and Applied Mathematics*	Mukherjee, A., Dey, O. & Giri, B.C.	2019	(Mukherjee et al.,2019) developed an imperfect inventory for integrated single-vendor, single-buyer where the vendor make investment to improve quality of items during production and same time the buyer offered price discount to consumers for backorder as incentive so that consumers could wait for some time to get their item. Their inventory management had followed continuous review by the buyer to place order of items in place of periodic review policy adopted by most of inventory management models. The lot size for an order depends upon lead time, backorders and lost sales.	Discussed investment to improve quality and price discount as incentive for shortage of items.

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					(Mukherjee et al.,2019) derived the optimal expected annual total cost of the integrated system using n-shipment policy.	

4. Research Gap

- The inspection process for imperfect quality production and imperfect inspection process in earlier researches had been conducted by the buyer after receiving a fresh lot of items. This research work tried to examine the impact on integrated inventory management if the inspection process has been conducted by the vendor along with the production of items with and without backorder.
- The buyer, sometime do not want to go for integrated inventory management and tried to optimize costs related to the buyers only. This is referred as the buyer's independent decision. For the buyer's independent decision, inventory management costs of the integrated and independent decisions have been compared.

5. Research Problem

Production schedule, production quantity, quality management, inventory/stock management, items shipment scheduling and meeting customer demands smoothly are some key operations of any organization. Better management of these operations helps an organization to reduce operational costs. Production of non-perishable items may be imperfect and items produced could contain some defective items despite of a number of quality controls measure adopted by the organization. All items undergo the inspection process to ensure that any defective item could not be sell in the market. The inspection process is conducted by human and there is always a chance of error. So the inspection process is also imperfect.

With an objective to reduce inventory costs, a number of researches were conducted to optimize above mentioned operation. Earlier researches assumed that the inspection process had been conducted by the buyer after receiving fresh lot of items. In this research it assumed that inspection process has been conducted along with production of items by the vendor in place by the buyer after receiving a fresh lot. This assumption is more realistic. This research is based on this problem and given a solution for without and with a shortage backorder.

6. Research Objective

- Identify the impact on the total cost of the supply chain management when the inspection is being performed at the vendor site for imperfect production quality and imperfect inspection process.
- Comparative analysis of vendor-buyer collaborative integrated model vs. the buyer's independent decision model.
- Perform Sensitivity Analysis of cost parameters and their impact on total expected cost of inventories management**.**

7. Assumptions / Hypothesis

This research is based on mathematical modeling. It has only assumptions for conversion into mathematical equations and solutions. For mathematical modeling following assumptions are made.

- The inventory model is for non-perishable items which has a long life span.
- It is single-vendor and single-buyer model.
- The meaning of buyer means the dealers that purchases items from a vendor and sells them to consumers of items.
- The rate of production of items "P" is greater than the rate of demand "D" i.e., P > D.

- Production lot is greater than supply lot and used to supply number lots to the buyer to meet demand. Number of lots “n” effects lot size and total cost of inventory. The optimal value of “n” is determined by the model.
- “T” is the time duration between two consecutive supplies to the buyer.
- The production process is of imperfect quality and produces some defective items with probability of “p”.
- 100% inspection of items has been conducted. The inspection process is also assumed to be imperfect and there are some errors during the inspection. An inspector may classify non-defective items as defective items (Type I inspection error) with probability e_1 or defective items as non-defective items (Type II inspection error) with probability e_2.
- The inspection rate “x” is greater than the production rate “P” i.e., $x > P$.
- Items classified as defective are disposed at discounted rate.
- Defective items B_2, which are classified as non-defective items by an inspector (type II inspection error), sold at market and later returned back by consumer under warranty, are sent back to the vendor and disposed at discounted rate.
- As there are some items being produced defective, B_1 additional items are produced for each lot items Q making it to $Q + B_1$ items.
- In this research work, first two models (without backorder – Integrated Model and The buyer’s independent decision model) do not allow a shortage of items in the inventory and next two models (with backorder – Integrated Model and The buyer’s independent decision model) allow a shortage of items with consent from the buyer that he/she will wait for fresh lot items to arrive.

8. Scope of the Research

- The research is for single vendor and single buyer.
- Only non-perishable items have been considered for this research work.
- The demand rate, production rate, percentage of defective items in production lot, inspection rate, type I and type II inspection errors are deterministic and known probability distribution.
- This research focuses on imperfect production quality items with imperfect inspection process.

9. Research Methodology

The research is based on mathematical modelling. Assumptions are made to represent the real life situation of inventory management. The inventory management situations have also been represented through figures. Using assumptions and figures inventory management related costs are translated into mathematical equations. This research has the following four models

- Integrated model where backorder has not been allowed
- The Buyers independent decision where backorder has not been allowed
- Integrated model where backorder has been allowed
- The Buyers independent decision where backorder has been allowed

Following are formulation of cost for the vendor

Inventory Carrying cost for the vendor

$$= n\left(Q+B_1\right)\left\{\frac{Q}{P}+\left(n-1\right)T\right\}-\frac{1}{2}n\left(Q+B_1\right)\frac{n\left(Q+B_1\right)}{P}-\left\{\frac{Q}{P}+\left(n-1\right)T-\frac{n(Q+B_1)}{p}\right\}nB_1-\frac{n\left(n-1\right)TQ}{2}$$

after solving

$= \frac{h_v}{2P}$ $[(2n - n^2)Q^2 + n(n\text{-}1)PTQ + n^2 {B_1}^2]$

Setup Cost $= S_v$

Warranty Cost $= n(Q + B_1)pC_w = \frac{npc_wQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npc_wQ}{A}$

Type I Error $= n(Q + B_1)(1 - p)e_1C_r = \frac{n(1-p)e_1C_rQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{n(1-p)e_1C_rQ}{A}$

Type II Error $= n(Q + B_1)pe_2C_{av} = \frac{npe_2C_{av}Q}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{npe_2C_{av}Q}{A}$

Inspection Cost $= n(Q + B_1)C_i = \frac{nC_iQ}{1-\{p(1-e_2)+(1-p)e_1\}} = \frac{nC_iQ}{A}$

Where $B_1 = \frac{p(1\text{-}e_2)+(1\text{-}p)e_1}{1\text{-}\{p(1\text{-}e_2)+(1\text{-}p)e_1\}}Q$

Total cost for the vendor $TC_v(n, Q)$ is

$$TC_v(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big]$$

Where $A = 1 - \{p(1-e_2) + (1-p)e_1\}$

Following are formulation of cost for the buyer

$$TC_b(n,Q) = K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{1-\{p(1-e_2)+(1-p)e_1\}} + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{[1-\{p(1-e_2)+(1-p)e_1\}]^2}Q^2$$

The total cost of the integrated inventory management without shortage backorder

$$TC_C(n,Q) = S_v + \frac{npc_wQ}{A} + \frac{n(1-p)e_1C_rQ}{A} + \frac{npe_2C_{av}Q}{A} + \frac{nC_iQ}{A} + \frac{h_v}{2P}\Big[(2n - n^2)Q^2 + \frac{n(n-1)(1-p)(1-e_1)PQ^2}{AD} + \frac{n^2\{p(1-e_2)+(1-p)e_1\}^2Q^2}{A^2}\Big] + K + nF + \frac{nc_{\alpha\beta}pe_2\,Q}{A} + \frac{nh_b}{2D}\frac{[1-\{p(1-2e_2)+(1-p)e_1\}]\,(1-p)(1-e_1)}{A^2}Q^2$$

There are probabilities for a defective percentage of items, type I inspection errors and type II inspection errors. For these probabilities, the theory of expectation has been used to get expected values. The costs have been calculated for one production cycle. Renewal and Reward theorem has been used to get expected costs of inventory management from one production cycle.

Numbers of shipments per order and economic order quantity (EOQ) of inventory management have been derived by optimization inventory management costs.

Earlier research works had been tested by a numerical example. During the literature review, it has been found that all these researches had been using the same numerical values for different parameters like demand, production rate etc. The equation for optimal total expected cost and economical order quantity for all models of this research have been calculated and found that these total expected costs are lower among earlier research works. Using these numerical examples, sensitivity analysis has been performed.

10. Data Analysis

This research is based on mathematical modeling and does not have any primary data. Mathematical models are based on assumptions. Numerical examples using commonly used numerical values (by earlier researches) generate some data values. Sensitivity analysis also generates data values. Analysis of these data has been done in the research.

11. Finding and Conclusions

Total expected costs with economic order quantity (EOQ) for integrated models without and with shortage backorder have been given in the table given below

Comparison of the integrated models

Values	Without backorder	With backorder
minimum Expected Total Cost (ETC) (in $)	2,01,226.23$	2,00,516.06$
Economic Lot Size (number of items)	769.4	919.8
Optimum Backorder quantity (number of items)	NIL	306.6

The above result shows that for the integrated inventory management where shortage backorder has been allowed is a better option.

Comparison of the buyer's independent decision models

Values	Without backorder	With backorder
minimum Expected Total Cost (ETC) (in $)	208459.45$	2,11,694.62$
Economic Lot Size (number of items)	1581.35	1224.59
Optimum Backorder quantity (number of items)	NIL	408.20

The above result shows that for the buyer's independent decision inventory management is not good when it is compared with the integrated inventory management. It also shows that allowing shortage backorder is not a good decision for the buyer independent decision inventory management. Whereas allowing shortage backorder is a good decision for integrated inventory management.

Sensitivity of the total expected cost with change in different parameters in inventory management has been analyzed in the thesis.

12. Contributions

The four mathematical models with and without shortage backorder for integrated and buyer independent decision has been derived successfully. These models considered for imperfect production quality, imperfect inspection process and inspection process done by the vendor along with the production of items. The total expected cost of inventory management of these models is below with respect to earlier similar models. Second, models are derived for the situation where the buyer takes independent decisions on economic order quantity. These models are compared with integrated models. Third, Analysis of outcomes of numerical outcomes gives insight of an inventory management which would help manufacturing industry to take correct decisions. For example, allowing a shortage of backorder in the buyer's independent decision is a wrong decision. But, allowing a shortage backorder in the integrated model is a welcome decision. For cutting down the total expected cost further, the sensitivity analysis will help the industry to find the areas where they could invest to get the maximum outcome of their investment.

Earlier research works on imperfect production quality and inspection error had assumed the inspection of items was conducted by the buyer after receiving items from the vendor. As mentioned in the research gap, in this research has focus on inspection process at the vendor site along with production of items and compares the expected total cost and found lower expected total cost. This outcome will help manufacturing industries to decide which place of inspection (the vendor site or the buyer site) will be beneficial for them. If they will go for inspection process at the vendor site then from four different models discussed in the research work will help them to take correct decision.

13. Limitations and Scope for future research

The efficiency of inspectors and their idle time have not been covered in this research and could be taken in future work. The Impact of inspection errors on the total expected cost is found very high in the research. The Impact of tanning and the use of advanced equipment would reduce the total expected cost significantly. Analysis of the use of tanning and advanced types of equipments has not been covered in the research and could be taken as further research work.

14. Keywords

Supply-Chain Management, Inventory Management, Expected Total Cost, Economic Order Quantity (EOQ), Imperfect Production, Imperfect Inspection

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